VALUATION

Monte Carlo Simulation: Assessing A Reasonable Degree of Certainty

By Christopher Daily, MBA; and David Solis, MSF, CVA, MAFF

In the world of valuating damages, particularly as it relates to projecting future financial outcomes or those that would have occurred but for a certain event, experts must grapple with achieving a reasonable degree of certainty. Whether it be projecting future earnings of an individual or forecasting revenue and expenses for a company, financial experts cannot—with one-hundred percent certainty—predict financial outcomes that have not occurred or did not occur. We, therefore, abide by the "reasonable degree of certainty" standard in applying our skills, knowledge, education, experience, and/or training to the facts of the matter to form opinions on those financial outcomes that never actually occurred.

This article explores the application of Monte Carlo simulation to damages analyses containing inputs that may not be known with a reasonable degree of certainty. It exemplifies the benefits as well as the drawbacks of Monte Carlo simulation through the analysis of two hypothetical scenarios: a simple lost profits analysis and a more complex construction delay claim requiring the evaluation of lost profits.

HOW DOES MONTE CARLO SIMULATION APPLY TO EVALUATION OF DAMAGES?

Monte Carlo simulation has many applications within a variety of fields. This article only focuses on its application as a method to incorporate probability distributions to the inputs of a damages analysis. In other words, we are going to allow our assumptions to fluctuate according to defined probabilities.

In general, a financial expert's damages analysis will contain inputs based on facts, evidence, and assumptions. These inputs ultimately drive the analysis and the resulting evaluation of damages. There are instances when a financial expert is given or concludes a range to be appropriate as an input. For example, in a lost profits analysis, the expert may assume the incremental/avoided cost percentage to be between twenty-five percent and thirty percent, based on the historical performance of the business. In such circumstances, it is common for a financial expert to simply use the average (27.5%) and present a single result for lost profits. It is also common for an expert to use both ends of the range (twenty-five percent and thirty percent) and present a range of lost profits.

The latter method can become complicated as more of these types of inputs emerge in the analysis. For instance, the expert may also determine lost sales to be between 400 and 600 units per month, the loss period to occur over three to four years, and the sale price to increase anywhere between three percent and seven percent per annum. Add in discounting to present value, and it becomes less intuitive to determine which inputs will produce the highest and lowest value of lost profits, for purposes of presenting a range.

Incorporating Monte Carlo simulation can be helpful in analyses such as these. Monte Carlo simulation allows you to incorporate such ranges into your analysis in the form of probability distributions. It then runs a specified number of simulations randomly selecting values within your specified range and/or probability distribution, and provides results regarding the outcome of those simulations. The results contain helpful information such as the mean and median of the simulations, a specified confidence interval of the simulations, among other outputs.

It should be understood, though, that Monte Carlo simulation is not simply a tool in which you can just drop in some numbers and produce meaningful results. Monte Carlo simulation requires a certain amount of statistical knowledge to be utilized correctly and to accurately interpret the results.

SIMPLE LOST PROFITS ANALYSIS[1]

We want to first demonstrate Monte Carlo simulation using a very basic and simple analysis. This hypothetical scenario involves a company claiming lost profits. The loss period is five to seven years. Lost revenue is between $50,000 and $150,000 per year. Saved expenses as a percentage of revenues is twenty percent to thirty percent. For our analysis, we assume a six-year loss period, lost revenues of $100,000 per year, and a saved expense percentage of twenty-five percent. The calculations without Monte Carlo simulation are shown on Schedule 1 and result in lost profits of $450,000.

SCHEDULE 1: SIMPLE LOST PROFITS ANALYSIS—NO MONTE CARLO SIMULATION

ASSUMPTIONS

	Low	High	Assumed
Loss Period (Years)	5	7	6
Lost Revenue Per Year	$50,000	$150,000	$100,000
Saved Expenses %	20%	30%	25%

ANALYSIS

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Total
Lost Revenue	$100,000	$100,000	$100,000	$100,000	$100,000	$100,000	$	$600,000
Saved Expenses	25,000	25,000	25,000	25,000	25,000	25,000	$	150,000
Lost Profits	$75,000	$75,000	$75,000	$75,000	$75,000	$75,000	$	$450,000

Next, we incorporate Monte Carlo simulation into our analysis. We use a program called @RISK which works with Microsoft Excel[2] to perform the same analysis, with the following modifications to our assumptions:

- We utilize a discrete probability distribution to describe the delay period of five to seven years. There is a 33.33% probability of each duration (33.33% probability of a five-year delay period, 33.34% probability of a six-year delay period, 33.33% probability of a seven-year delay period).
- We utilize a uniform probability distribution to describe lost revenue per year. There is equal probability the lost revenue is anywhere between $50,000 and $150,000 per year.

1 This hypothetical scenario is not intended to represent a comprehensive analysis. It is intended to be overly simplified and vague regarding certain details, for demonstrative purposes.

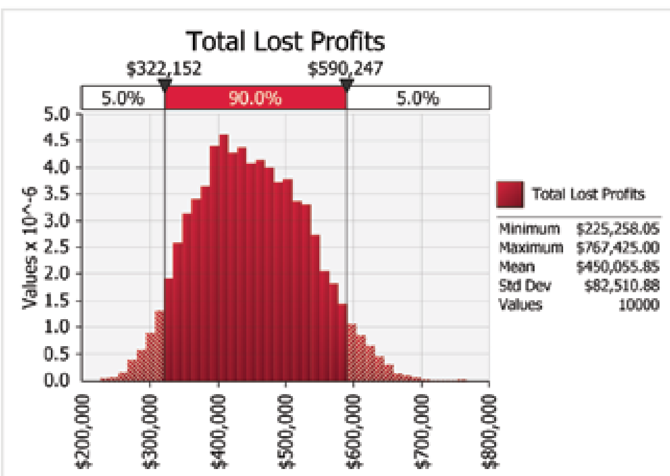
2 @RISK was developed by Palisade Corporation. It attaches to Microsoft Excel.

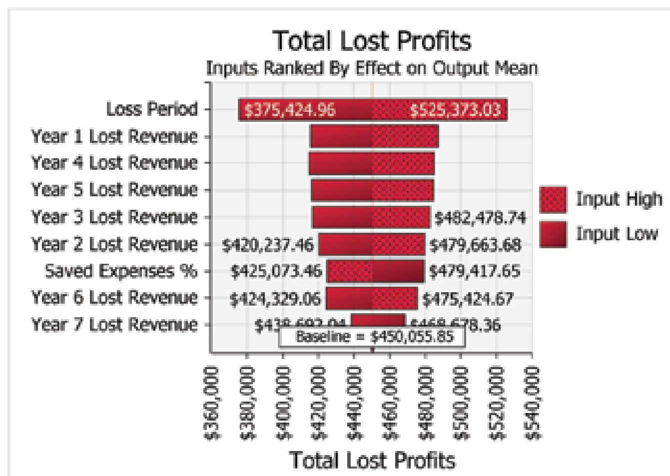
- We utilize a uniform probability distribution to describe the saved expense percentage. There is equal probability the saved expense percentage is anywhere between twenty percent and thirty percent.

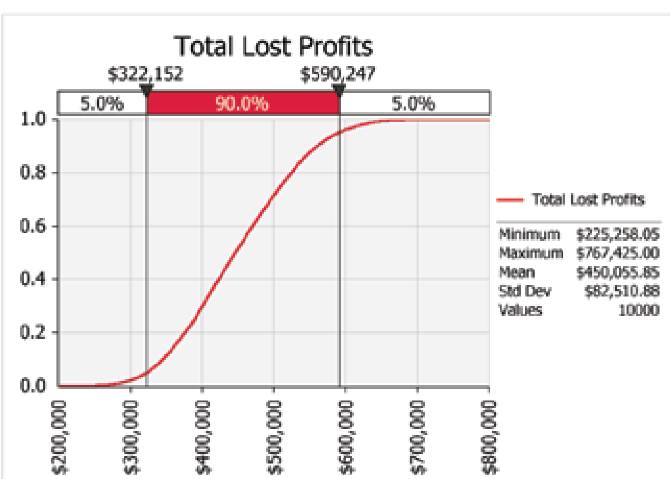
We ran the simulation 10,000 times, meaning @RISK performed 10,000 separate iterations of the analysis, randomly altering our inputs each time in accordance with the probabilities defined. The lost profits calculations using Monte Carlo simulation resulted in a mean of $450,056 and a median of $445,855 (@RISK output is shown on Schedule 2). Our results are approximately equivalent to the $450,000 result obtained from our analysis without Monte Carlo simulation.

So what was the point of doing Monte Carlo simulation for this analysis? Our resulting lost profits value is approximately equivalent to just multiplying lost profits of $75,000 per year (lost revenue of $100,000 minus saved expenses of $25,000) by six-years. Instead of going from Point A to Point B, we complicated the analysis and went from Point A, to Point M, to Point C, just to get to Point B. Let's look at some of the output we obtained from @RISK on Schedule 2.

SCHEDULE 2: @RISK OUTPUT REPORT FOR SIMPLE LOST PROFITS ANALYSIS







Although our ultimate opinion regarding the value of lost profits may be unaffected by using Monte Carlo simulation, we gain insightful information from having performed the simulation. For instance, we know our ninety percent confidence interval for lost profits given the variability of our assumptions, is $322,152 to $590,247. We can also see the relative effect of our variable inputs on the value of lost profits. Regarding our ultimate opinion regarding the value of lost profits, we can say we simulated the analysis 10,000 times allowing our variables to fluctuate, and the most likely outcome is lost profits of approximately $450,000. How does this affect our degree of certainty regarding our opinion? We argue that it strengthens it considerably.

CONSTRUCTION DELAY CLAIM[3]

We next look at a hypothetical scenario that involves more complexities. This hypothetical scenario involves a company which is claiming damages resulting from a construction delay. The company develops raw land into residential lots. It contracted with other entities to develop one hundred lots. The original completion date was July 1, 2017, and this completion date remained unchanged prior to the event causing the construction delay. The damaging event caused a construction delay of anywhere between three and seven months. The company expected to sell fourteen to eighteen lots per month prior to the delay, and still expects the same absorption given the delay. The sale price per lot is $100,000 as of the date of valuation. The short-term increase in sale price is estimated to be ten percent per annum, and is not expected to be below five percent or above twelve percent. Soft costs are fixed at $15,000 per month and continue until all lots are sold. The discount rate is twenty percent and the valuation date is May 1, 2017.

We have already evaluated past lost profits along with any other elements of damages. Our task now is to determine future lost profits due to the delay.

We utilize the facts and assumptions above to first perform the analysis without Monte Carlo simulation. Our analysis contains the following assumptions regarding the information above:

- The delay period is assumed to be five months (average of three to seven months).
- Market demand for the lots is sixteen lots per month (average of fourteen to eighteen lots per month).
- The annual growth rate of the lot prices is ten percent (expected to be ten percent).

The calculations are summarized on Schedule 3. The following are the results:

Present Value of Profits *But For* the Delay	**$9,542,846**
Present Value of Profits *Given* the Delay	**- 9,135,071**
Lost Profits	**$ 407,775**

Next, we utilize the facts and assumptions above to perform the analysis using Monte Carlo simulation. We again use @RISK to perform our analysis. Our analysis is identical to the one without Monte Carlo simulation above, apart from the following changes to the assumptions:

- We utilize a discrete probability distribution to describe the delay period of three to seven months. There is a twenty percent probability of each duration (i.e., twenty percent probability of a three-month delay period, twenty percent probability of a four-month delay period, etc.).
- We utilize a discrete probability distribution to describe the market demand during each month. We incorporate the fourteen to eighteen lots per month absorption rate, assuming each whole number absorption rate has a twenty percent probability during each month (i.e., twenty percent probability of a fourteen lots/month absorption rate during a specific month, twenty percent probability of a fifteen lots/month absorption rate during a specific month, etc.).
- We utilize a triangular probability distribution to describe the annual growth rate of lot prices. We define the expected value as ten percent, the minimum value as five percent, and the maximum value as twelve percent.

To demonstrate the use of Monte Carlo simulation, we ran this analysis at different levels of iterations. Using @RISK, you can choose the number of iterations you want the simulation to run. The most common number we have seen in practice is 10,000 iterations. We ran the analysis at 10; 100; 1,000; 10,000; 100,000; and 1,000,000 iterations to demonstrate the effects on the output (for detailed schedules and charts, please go to http://www.NACVA.com/VE-MayJune17.) The following is a summary of the present value of lost profits at the different levels of iterations of the analysis:

3 This hypothetical scenario is not intended to represent a comprehensive analysis. It is intended to be overly simplified and vague regarding certain details, for demonstrative purposes.

SUMMARY OF PRESENT VALUE OF LOST PROFITS

				90% Confidence Interval
Iterations	Mean	Median	Lower Bound	Upper Bound
10	$450,289	$417,279	$218,448	$690,273
100	$440,116	$410,847	$244,377	$690,199
1,000	$441,224	$434,258	$243,208	$665,770
10,000	$441,087	$434,916	$243,098	$665,935
100,000	$441,182	$435,830	$242,496	$667,619
1,000,000	$441,153	$435,526	$242,237	$667,711

You will notice as the number of iterations increases, we see convergence amongst each statistic. There are many insightful statistics on Schedules 4 through 9 (see http://www.NACVA.com/VE-MayJune17 for details) such as the relative effects of each variable input on the present value of lost profits. @RISK provides many other helpful statistics, but this article does not dive into all its capabilities.

In our analysis, without using Monte Carlo simulation, we determined the present value of lost profits to be $407,775. Using Monte Carlo simulation in our analysis, results in present value of lost profits of $441,153 using the mean, and $435,526 using the median (using 1,000,000 iterations). What is our opinion regarding the present value of lost profits? One methodology takes an average or expected value of each assumption, inputs those single assumptions into one analysis, and calculates one result. The other uses a range of values with associated probabilities for each assumption, runs multiple iterations of the analysis changing each assumption in accordance with its range and probability distribution, calculates multiple results, and provides the mean and median (among other statistics) of those results. Both are acceptable and provide valid results. Yet there is a material difference between the results of each methodology.

CONCLUSION

We used Monte Carlo simulation to redefine assumptions in damages analyses, from single constants to ranges with probability distributions. Through two examples, we saw that using Monte Carlo simulation can have a material effect on the ultimate outcome or no material effect at all. Though in both occurrences, we were provided with helpful statistics regarding the possible outcomes of analyses.

In circumstances where you can utilize single inputs as your assumptions with a reasonable degree of certainty, it may be more appropriate not to incorporate Monte Carlo simulation to determine resulting damages. Conversely, if you are unable to isolate your assumptions to a single input, Monte Carlo simulation may be the appropriate approach. Our analyses and ultimate results rely on our assumptions. If we cannot be reasonably certain about our assumptions, we cannot be reasonably certain about our results.

Regardless of which methodology you ultimately rely upon, we suggest that Monte Carlo simulation at least be considered as a tool in such analyses. It can provide helpful insight into your analysis if you have inputs or assumptions which cannot be identified as a single input with a reasonable degree of certainty. It can shed light on possible outcomes and therefore, it can be helpful in assessing your reasonable degree of certainty regarding the outcome upon which you rely. VE

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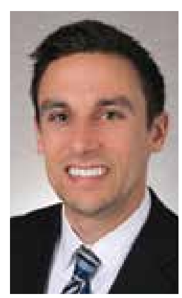

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